Concerning the Laws of Contradiction and Excluded Middle
by V. J. McGill,
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I

Tradition usually assigns greater importance to the so-called laws of thought than to other logical principles. Since these laws could apparently not be deduced from the other principles without circularity and all deductions appeared to make use of them, their priority was considered well established. Generally, it was held that the laws of thought have no proof and need none, that as universal constitutive or transcendental principles they are self-evident. There have been many dissenting opinions, of course, and many impressive systems erected upon a deliberate violation of these laws. Hegel, for example, held all three in contempt. The law of identity is empty and meaningless, apparently more empty and meaningless than any other proposition. The law of contra- diction is contradictory, while the law of excluded middle, as usually interpreted, is "so insignificant that it is not worth stating." On the other hand, one of the most basic doctrines of the Hegelian philosophy was that reality cannot be contradictory, which is simply one interpretation of the law of contradiction. In a certain sense Hegel took the law of contradiction more seriously than those contemporary philosophers, who, while decrying his dialectic as madness, reduce the principle to an arbitrary verbal convention. The usual objections to the laws of thought that [197] they are abstract and meaningless, that they are static and inconsistent with change, that they are psychological limitations or verbal conventions do not represent the majority opinion, which has held them to be prior to, and hence, more important than other logical principles. The traditional attitude toward these so-called laws of thought was bound to change with the development of the postulational method, just as the geometers' outlook was changed by the appearance of non-Euclidian geometries. Mathematicians entering the field without the accustomed reverence for logical precedents, constructed postulate sets in which the three laws did not occur. Not only were they unnecessary as premises, but it appeared in Whitehead and Russell's *Principia Mathematica* that they could be deduced from other logical principles, and could therefore lay no claim to priority. This opinion which is widely held today is a complete reversal of the traditional view. The laws of thought are now generally assigned the same status as other logical principles. One prominent school of thought, for example, reduces them all to tautologies. As Wittgenstein put it: "All these propositions say the same thing. That is, nothing." Although this is the dominant view at the present time, it seems in view of the following considerations, very doubtful. The fact that the laws of thought do not appear among the formal premises of a system does not mean that they are not involved in the proofs. "This problem," Johnson says, "is perhaps of purely technical interest, and the attempt at its solution presents a fundamental, if not insuperable, difficulty: namely, that the procedure of deriving new formulae from those which have been put forward as to be accepted without demonstration, is governed implicitly by just those fundamental logical principles which it is our aim to formulate explicitly. We can, therefore, have no assurance that, in explicitly deriving formulae from an enumerated set of first principles we are not surreptitiously using the very same formulae that we propose to derive. If this objection cannot be removed, then the supposition that the whole logical system is based upon a few enumerable first principles falls to the ground." (Vol. I, p. 223.) [198] The importance of *Principia Mathematica* owed much to the fact that it put down on paper more of the premises required for its proofs than any comparable system had ever done (with the exception of Frege's logic), but as Johnson suggests, there were still many loopholes for intuitive steps and concealed premises. In logic, as in no other science, the rules of inference employed in proving theorems are among the theorems to be proved. Since rules of inference such as: "What follows from a true proposition is true" are not put down as premises in the arguments, and cannot be, as Lewis Carroll demonstrated, there is always a possibility that in proving a proposition we are using the same principle, or one logically dependent upon it, as a rule of inference. Thus the rule of inference, "What follows from a true proposition is true", is used to prove the theorem, *3.35, which is the same thing in symbolic form. In general, the rule of inference insures the assertion of non-self-contradictory propositions. If what followed from a true proposition were ever a false proposition, then self-contradictory propositions would occur in the system. The rules of substitution, which are not explicitly stated in *Principia Mathematica*, also open up the possibility of this kind of circularity in the proofs. For example, if a proposition p occurs more than once in a proposition we are proving, we are allowed to substitute ~p for every occurrence of p. But we are not permitted to substitute ~p in one place unless we also substitute ~p in every other place in which p occurs. Thus, in $p \equiv p$ (p is materially equivalent to p) we can substitute ~p for p in both places but not in one place only, for in that case we should have $p \equiv \sim p$, which contradicts the definition of equivalence. It is clear that the rules of substitution serve to exclude violations of the law of contradiction and the law of excluded middle as well. The rule of inference, likewise, not only enables us to drop our premises but to exclude self-contradictory propositions. Definitions in *Principia Mathematica* also merit close attention since, as the authors themselves point out, they often convey more important information than is contained in the theorems. [199] "A definition," Russell says, "is concerned wholly with the symbols, and not with what they symbolize. Moreover, it is not true or false, being the expression of a volition, not a proposition. (For this reason definitions are not preceded by the assertion sign.) Theoretically, it is unnecessary ever to give a definition: we might always use the definiens instead, and thus wholly dispense with the definiendum." (*Principia Mathematica*, vol. I, First Edition, p. I2.) He also remarks that he does not need to define definition or to introduce it as a primitive idea because definitions are "no part of our subject, but are, strictly speaking, mere typographical conveniences." Three comments can be made at this point. First, since definitions convey information and give clarity and definiteness to expressions, as Russell claims, they would seem to make an assertion; and thus to be true or false. Russell seems to conclude that because they do not make assertion about the subject matter, i.e., elementary propositions, they do not make assertions at all. It is more reasonable to suppose with G. E. Moore that what they assert is that in *Principia Mathematica* a certain expression, the definiendum, will be used as short for another, more analytical expression, the definiens. In this sense the definition is true if the authors are consistent. Russell's assertion that definitions are mere typographical conveniences and that the avoidance of cumbersome expressions and complications is their only service is also, of course, very doubtful, in view of the fact that the transition to all new topics is effected by definitions. Instead of starting off on a new subject with new axioms, as Peano does in his articles in the *Formulaire de mathematiques*, Russell begins with definitions. This procedure, while very useful, leaves open the possibility that new axioms are being introduced in the guise of definitions, and that *Principia Mathematica* contains more primitive propositions than the authors believe. A definition is merely verbal, Russell says, and no part of the subject, and yet, by means of the theorem, $\vdash \cdot p \supset p$, any definition can be transformed into an equivalence, into an asserted proposition of the system, into a proposition not about words, but about things. Definitions are often of crucial importance. Without the definition [200] of conjunction, $p \cdot q \equiv \sim(\sim p \vee \sim q)$ Df., which is one form of DeMorgan's principle, Whitehead and Russell's deduction of the law of contradiction from the law of excluded middle could not have been accomplished, nor could their system have retained the same degree of duality and completeness. It will be seen in what follows that the definition of conjunction, like the definition of material implication, leads to important paradoxes, which is another indication that the definitions of *Principia Mathematica* are not mere typographical conveniences unrelated to the subject, and that they are not as innocent as Russell claims. These considerations are not new, but they are seldom, I think, applied to the point at issue. Taken together they are sufficient to disprove the notion that the laws of thought can without circularity be deduced from other principles. At the same time, they argue for the priority of the laws of thought. An examination of the truth-table method of proving propositions of *Principia*

Mathematica reinforces this conclusion. Those acquainted with the writings of Post, Wittgenstein and others who have followed this method will understand that the various logical constants such as implication, disjunction, etc. may be defined in terms of the truth possibilities which they allow. Implication can be defined as holding for every pair of propositions, p q, except when p is true and q false, disjunction, when they are not both false, while conjunction only holds when p and q are both true. With the logical constants defined in terms of true and false possibilities, we can test the truth of all of the elementary propositions. When the truth tables are set up, however, two meta-logical principles universally govern the possibilities. One prevents the same proposition p from having both a true and a false sign in its several occurrences and the propositions p and ~p, when they occur in the same proposition, from having the same sign. The other insures that every proposition has a sign, either true or false. The first principle, of course, is the law of contradiction, while the second is the law of excluded middle. Both are necessary for the proving of the elementary propositions of *Principia Mathematica* by the truth-table method. This is [201] rendered even clearer by the example of the law of contradiction itself. The law is proved in *Principia Mathematica* by the law of excluded middle, De Morgan's principle and "Identity", and many readers may not realize that another unstated principle is involved, namely, the law of contradiction itself. When the truth-table method of proof is used, however, everyone can see. Only when the possibility of p and ~p having the same truth value is excluded, can the conclusion, ~(p and ~p), be demonstrated. Here, in other words, the law of contradiction is used to prove itself. That a kind of circularity is also involved in the Principia proof of the law of contradiction was argued above on the basis of general considerations. First, there is the fact that the rule of inference is used to exclude self-contradictory propositions, and to include consistent ones. This principle is involved in the proof of the law of contradiction. Secondly, the proof makes use of another principle. If p occurs more than once in a proposition (which is the case with almost all of the elementary propositions of *Principia*) the same substitutions must be made in each case. The rule employed here is a variant of the law of contradiction. It seems fairly clear that the sense in which the law of contradiction and the law of excluded middle can be deduced in contemporary logic from other logical principles is a very technical one. The rules of inference, substitution, the definitions, and the truth-table proofs all furnish evidence. In the truth-table proofs of the two-value logics, for example, the laws of contradiction and excluded middle are always employed, and few others are needed. If one were to prove that these two principles have no priority or special importance, it would be necessary to set up a two-value logic in which they are not prior, either formally or informally. This, it seems to me, impossible to do. In the truth-tables it would always be necessary to allow p in its various occurrences only one value, or sign, and to rule that p and ~p have different values and that all elements have some value or other. The laws of contradiction and excluded middle would be necessary here and no other principles by themselves would serve the purpose. This seems to imply that these two laws are prior and more important. It is therefore difficult [202] to understand why Cohen and Nagel say that "Even if other principles of logic could be derived from the traditional three, that would not make these more important, or more certain, than any of the others." (*Logic and Scientific Method*, p. 182.) "Importance" is a relative term. In relation to deductive systems, one would suppose "importance" could only mean priority or deductive power.

II

Closely bound up with the theory that the laws of thought are prior to the other principles is the much disputed question of their dependence or independence *inter se*. Aristotle,[1] and many others put the law of contradiction and the law of excluded middle in a single formula, p or not-p, disjunction being understood in its exclusive sense. So interpreted, this formula states that p and ~p are not both true and are not both false, and accordingly, states the law of contradiction as well as the law of excluded middle. Yet it would seem easy enough to distinguish between them, and Aristotle and most of his followers have held not only that the two

[1] *Categoriae* 10, 13a, 37.

laws are distinct but that they are independent, that neither implies the other. In modern two-value systems, on the other hand, the one is regularly de- rived from the other and definitions, axioms or rules of substitution, are adopted which make it impossible to assert the one without implying the other. Thus in the Boole-Schröder logic the law of excluded middle follows directly from the law of contradiction with the help of De Morgan's principle and the same is true in Lewis and Langford's version of the system of material implication (*Symbolic Logic*, p. 136) and in Lewis's system of strict implication, where the same proof is used. In *Principia Mathematica* itself, the procedure is different. Since disjunction, rather than conjunction, is taken as primitive, the order of the laws is reversed and the law of excluded middle is used to prove the law of contradiction. Likewise, since ~~p in a proposition cannot be replaced by p, the proof is also longer, and involves many more assumptions. It involves at least the following: The [203] definitions of implication and conjunction and the rule of inference, the procedure of substitutions and three primitive propositions (*I.2, *I.3, *I.6). If, however, ~~p can be replaced by p, the one law follows from the other directly by means of De Morgan's principle. Thus in Boolean algebra or Lewis and Langford's version of the system of material implication, the law of excluded middle follows directly from the law of contra- diction with the help of De Morgan's principle, the principle of double negation being assumed in the procedure of substitution. Whether disjunction, conjunction or even Sheffer's stroke function is taken as primitive in a given system, is optional and whether the principle of double negation is used in the substitutions or not, is a matter of convenience. The important thing is that, if the usual assumptions are made, the law of contradiction and the law of excluded middle are equivalent. That is, either both are true or both are false. There is, of course, nothing surprising in the fact that two theorems in the systems we are discussing should be equivalent. Any two theorems of the first section of *Principia Mathematica*, for example, are equivalent. What is surprising is that precisely these two theorems, the law of contradiction and the law of excluded middle, should be equivalent, for an important body of opinion from Aristotle on has held that they are independent, and, more particularly, that the law of contradiction can be asserted without asserting the law of excluded middle while the latter can be denied without denying the former. Thus in the *Metaphysics* (Γ, 3), Aristotle describes the law of contradiction as "the most certain of all principles," "a principle which every one must have who understands anything that is" and "which every one must know who knows anything." His attitude toward the law of excluded middle is very different. Although in the *Metaphysics* (Γ, 7) he apparently wants to prove the law of excluded middle from the definitions of true and false, assuming, of course, the law of contradiction, in another place (*De Interpretatione*, Ch. 9, 19a) he argues that the law of excluded middle does not apply to judgments about the future. With respect to what is actual, the law holds good, but where indetermination enters, as in the case of things [204] in the future, it fails. "It is therefore plain," he says, "that it is not necessary that of an affirmation and denial one should be true and the other false (determinately). For in the case of that which exists potentially, but not actually, the rule which applies to that which exists actually does not hold good." This argument of Aristotle was revived some years ago by C. D. Broad in *Scientific Thought* and more recently by J. Lukasiewicz.[2] Its logical force does not concern us here.[3] What interests us in this connection is that Aristotle is ready to reject the universality of the law of excluded middle while asserting that the law of contradiction is the most fundamental and indubitable of all principles. For we have seen that if one law is asserted, the other must be asserted, unless of course, De Morgan's principle or the principle of double negation (~~p ≡ p) is rejected. But Aristotle, it is clear, accepted both these principles. Thus in *Metaphysics*, Ross Edition, (K. 1063b) he states that: "No intermediate between contraries can be predicated of one and the same

[2] "Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalkuls" in *Comptes Rendus des Séances de la Société des Sciences et des Lettres Varsovie*, Classe III, Vol. xxiii, 1930. Fascicule 1-3, pp. 51-77.

[3] See "On Mr. Broad's Theory of Time" by R. M. Blake, Mind (1925) for a discussion of Broad's view, and "Are Some Propositions Neither True Nor False?" by Charles A. Baylis *Philosophy of Science* (April, 1936) for a criticism of Lukasiewicz.

subject, of which one of the contraries is predicated. It the subject is white we shall be wrong in saying it is neither black nor white, for then it follows that it is and is not white; for the second of the two terms we have put together (black and white) is true of it, and this is the contradictory of white." What Aristotle says here is that if x is white (w) and x is not (black or white) then x is white and not white. This only follows, however, if x is ~(b or w) implies x is (~b and ~w). In other words, the argument assumes a variation of De Morgan's principle: ~(b or w) ≡ ~b and ~w. Although De Morgan's principle was not explicitly stated until the nineteenth century, it was utilized long before. When Aristotle and Leibniz state that p or ~p (where disjunction is taken in the exclusive sense) [205] states the law of contradiction as well as the law of excluded middle, he is assuming De Morgan's principle. Likewise, in the course of the long controversy between the upholders of exclusive disjunction and the defenders of non-exclusive disjunction, the latter naturally maintained that p or q is true except when both p and q are false, while the former contended that p or q is true except when p and q are both false or both true. In general it appears that "neither A nor B" admits of two interpretations, namely: "not (A or B)" and "(not A and not B)". Common language passes from the one to the other with- out a pause, and accepts De Morgan's principle in this form automatically. The acceptance of the principle of double negation is also natural and inevitable. Both principles are intuitively certain, and also required for a form of duality characteristic of modern logical systems. If either is omitted drastic alterations in logical systems would be necessary. Yet if both are retained, and the usual assumptions are made, the law of contradiction and the law of excluded middle are obviously equivalent, a thing which most logicians, from Aristotle on, have denied at least by implication. If the principle of double negation is taken in its usual form, ~~p ≡ p, the whole matter, as we have seen, is much simplified. In this case, the law of contradiction and the law of excluded middle are equivalent if only De Morgan's principle and double negation are granted. In either case the denial of the law involves the denial of the other. But many of the people who are prepared to eliminate the law of excluded middle (or to restrict its generality) have no idea of abandoning the law of contradiction or double negation, and have no objection, so far as I know, to the very obvious principle of De Morgan. On the contrary. They cling to the law of contradiction as tenaciously as the rest of us, and no general interest is evoked by the idea of creating a system which would dispense with it. They see advantages in excluding the one law, but not the other. The enemies of the law of excluded middle hail from different disciplines and employ different arguments. There are those who argue that classifications in the social sciences which violate [206] the law of excluded middle are often more useful than those which do not (Dewey); there are the mathematicians in the circle of Brouwer, who contend that the best way to avoid the paradoxes of Mengenlehre is to restrict this law; and finally, there are the philosophers, such as Broad, who think the law stands in the way of any adequate explanation of the fact of change. None of them show any tendency to question the law of contradiction. Yet, as we have seen, the two laws stand or fall together if the customary assumptions are made. The dialecticians, on the other hand, in so far as they deny both laws, and not merely one, would appear to be in a better position. It may be, of course, that our intuition that the two laws are logically independent is erroneous, that they are, in fact, (materially) equivalent. When one adopts this alternative one is caught on one horn of the dilemma. If the two laws are equivalent, the violation of one involves the violation of the other. The assertion that a certain man is neither bald nor non-bald implies that he is both. Aristotle's plausible (if false) contention that judgments about the future are neither true or false entails the dubious proposition that judgments about the future are *both* true and false. Broad, E. T. Bell and others, who find the rejection of the law of excluded middle possible and attractive, are thus obliged if our argument is correct, to reject the law of contradiction as well, a result which neither would welcome. In the same way the contention that certain propositions such as: "All numbers of the form 2^{2n+9} + 1 are factorable" are neither true nor false implies, given the common assumptions, that all propositions of the kind are both true and false. This is one horn of the dilemma. If the law of excluded middle and the law of contradiction are equivalent in any system, inadmissible consequences follow. The other horn of the dilemma is reached if the other course is taken, if the equivalence is denied. In this case, as we have seen, a contradiction develops. Just as Aristotle became involved in a contradiction in so far as he restricted the law of excluded middle (while upholding the law of contradiction and assuming De Morgan's principle and $\sim\sim p \equiv p$, so modern logicians and philosophers, if they make the same assumptions, are in the same position. The only way [207] out of this contradiction would appear to be the elaboration of a workable system of logic in which the law of contradiction is retained, while law of excluded middle, and either De Morgan's or $\sim\sim p \equiv p$ or some alternative principles, is omitted. It might appear that in *Principia Mathematica* the simple omission of one of the three primitive propositions mentioned above (*I.2, *I.3, *I.6) would remove the paradox, since in this case $p \supset p$ could not be proved. But the prospects are not encouraging. If $p \supset p$ could not be proved, some other way would have to be found for making use of the definitions, and this would involve great changes. It would be interesting, however, to have such a system developed. The paradox we have been discussing could be discovered, I think, in most of the logic since Aristotle. It becomes clearer in modern formal systems. In *Principia Mathematica* it has the same status as the paradoxes of implication, such as, for example, the theorem that a false proposition implies all propositions. It does not arise from the formal system itself, but from the formal system in relation to our common understanding and employment of logical operations. The question now arises whether three-value and multi-value logics do not resolve the paradox. For it is commonly held that there are multi-value logics which assert the law of contradiction and omit the law of excluded middle. If this were so, it would meet the requirements, since the paradox would not appear in a system in which the law of contradiction did not imply the law of excluded middle. Unfortunately, it is extremely doubtful that such a system exists. In the Lukasiewicz-Tarski three-value system, for example, the law of excluded middle does not hold, but neither does the law of contradiction. Both are doubtful (2) when both p and ~p are doubtful (2). Far from resolving the paradox, this three-value system of Lukasiewicz-Tarski seems to add weight to the contention that the two laws, when the usual assumptions are made, are inextricable. In order to retain De Morgan's principle, double negation and other needed propositions, negation, disjunction and conjunction are so defined as to exclude not only the law of excluded middle, but also the law of contradiction. Strictly speaking, of course, it is not these two laws but rather their analogues which fail in the three-value logic. [208] It would be more correct (but also more cumbersome) to speak of the "analogues" of the principles of the two value logic holding or failing in the three-value logic. The Heyting logic[4], which gives formal expression to the ideas of Brouwer, appears to come much closer to solving our paradox. For in this system the analogue of the law of contradiction can be proved, while the analogue of the law of excluded middle cannot. It is interesting to observe, however, that this result is only achieved by abandoning the principle of double negation (or rather, its analogue) with the consequence that new paradoxes arise. For example, the system disqualifies not only some, but all indirect proofs, even the common proofs of plane geometry. (Moreover, there are complications. The Heyting logic needs an unusually long list of primitive propositions, each logical con- stant must be introduced as a separate primitive idea, and the duality which distinguishes modern logical systems, is much reduced.) Since the truth-value interpretation of Heyting's system leads to paradoxes and there are other interesting interpretations of it which do not, the propriety of this interpretation could be questioned. When it is also remembered that the laws of two-value logic do not occur in Heyting's system, but only their analogues, it appears impossible to hold that this system excludes the law of excluded middle while retaining the law of contradiction. We are therefore obliged to conclude that in no system with

[4] A. Heyting, "Die formalen Regeln der intuitionistischen Logik," *Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalish-mathematische Klasse*, I930, pp. 47-58. Also see Orrin Frink's interesting discussion, "New Algebras of Logic," *American Mathematical Monthly*, Vol. XLV, No. 4, April, I938.

which we are acquainted are the two laws independent. In every logic in which the law of contradiction occurs the law of excluded middle is also asserted, while the denial of the latter involves the denial of the former. But, as we have seen, this conclusion contradicts the common view.

III

The equivalence of the law of contradiction and the law of excluded middle (assuming De Morgan's principle and $\sim\sim p \equiv p$ [209] is of some interest in relation to the dialectic. If these laws are equivalent the restriction of one is as reasonable as the restriction of the other, and dialecticians can scarcely be blamed for rejecting not one, but both. Indeed if one is denied, the other must be denied. It follows, if our reasoning is correct, that the present philosophers and logicians who are ready to restrict the law of excluded middle must be prepared to restrict the law of contradiction as well, or to work out a new system which evades the paradox. The question whether the law of excluded middle applies universally seems to depend upon the question whether dichotomies occur universally in the world. The law is valid when, and only when, it is possible to find a mathematical point dividing a process of change into a and non-a, or to make a Dedikind cut dividing the continuum into two segments a and b such that every element or phase in a is less with respect to the quality which is changing than any element in b and every element or phase of b is greater in the same sense than any element of a. That dichotomies or Dedikind cuts are to be found everywhere in society and nature has been questioned or denied by many writers, and there is perhaps no need to repeat their arguments and instances. Professor Dewey in his recent Logic puts the position very strongly: "The notion that propositions are or can be, in and of themselves, such that the principle of excluded middle directly applies is probably the source of more fallacious reasoning in philosophical discourse and in moral and social inquiries than any other one sort of fallacy ... An example sometimes put forward to show the meaninglessness of the principle of excluded middle is its inapplicability to existences in process of transition. Since all existences are in process of change it is concluded that the principle is totally inapplicable. For example, of water that is freezing and of ice that is melting, it cannot be said that water is either solid or liquid. To avoid this difficulty by saying that it is either solid, liquid or in a transitional state, is to beg the question at issue: namely, determination of the transitional intermediate state. The objection is wholly sound on any other ground than that the canon expresses a condition to be satisfied." The law of excluded middle, on this interpretation, is not an ontological principle but only heuristic. Since we cannot know [210] in advance of inquiry that any two propositions, p and ~p, contradict each other, we cannot know in advance that the law of excluded middle holds for all propositions. And for the same reason we cannot know in advance that the law of contradiction holds for all propositions Professor Dewey concludes accordingly that it is also impossible to interpret the law of contradiction ontologically. This is what the argument in this paper would lead us to expect. If the two laws are equivalent, granted the usual assumptions, then the one could not lose ontological status without the other losing it too. The status of the two laws would appear to be the same. Both are used as heuristic principles. Both specify conditions to be satisfied, and both are sometimes satisfied and sometimes, not. Indeed, if our contention is correct, the conditions which satisfy one law, must also satisfy the other (i.e. when the usual assumptions are made). But it is precisely at this point that it becomes difficult to understand Professor Dewey's account. Although he recognizes instances in which the law of excluded middle does not apply, as, for example, the case of biological species, he fails to give examples where it does apply. Although he recognizes the relevance of the laws to the world, he appears to hold that the conditions they set up to be satisfied either cannot be satisfied, or have not up to date been satisfied, in any instance. It is also not clear what the status of the laws would be if the conditions they set up to be satisfied were satisfied in a given field. Would the laws restricted to this field remain mere guiding or heuristic principles, or would they not also describe and correspond with the facts of this field? One possible view of this matter would be that the laws of contradiction and excluded middle are satisfied when, and only when, a strict dichotomy or Dedikind cut can be set up in the processes of nature and society, and that where this is not possible a certain range of propositions would have to be admitted which are neither true nor false, and both true and false. These propositions could be eliminated, of course, by a definition of "proposition" requiring that they satisfy the laws of logic, but it is doubtful if anything could be gained by a mere change of [211] name. We should still be in doubt a great deal of the time whether our expressions answer to the customary laws of logic, whatever we call them. In practice, of course, the amenities of polite or routine discourse oblige the assumption that non- Aristotelian propositions do not occur. But when contention or original thinking begins, dichotomies are challenged and the assumption breaks down. The process of denying dichotomies and establishing new dichotomies is a process of clarification. It is subjective or Socratic dialectic which few would question. The customary attitude to objective dialectic, to dialectic as a process in nature and society is utterly different. The idea that dialectical, non-Aristotelian propositions could correctly describe the developmental processes of nature and society is a *bete noire* to many writers who, oddly enough, congratulate themselves on having overcome the Aristotelian view of the world, and who often regard the Aristotelian rules of logic as mere verbal conventions. It would be absurd, of course, to hold that nature imitates our subjective processes of clarification. On the other hand, it seems very arbitrary to deny in advance of inquiry, and in the teeth of much evidence, that sequences of dialectical, non-Aristotelian propositions can describe the transitional states of the objective world. The hypothesis that Aristotelian logic applies in so far as it is possible to set up dichotomies, and that some non-Aristotelian or dialectical logic applies in so far as this is impossible, provides at least a basis for further inquiry.

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