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"Once more on the paradox of movement, on dialectical and formal-logical contractions."

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The paradoxes of movement are the four antinomies of Zeno, presenting an analysis of movement. Among these the antinomy of the "arrow" is usually emphasized, evidently because it is directed at on the problem of the essence of movement to the highest degree and shows the basic difficulties that are connected with understanding this essence.

It is well known that preceding from ostensively true claims about it, in each instant of the flight of the arrow it is found in a definite place in space and that time is composed of moments (points of time, without duration), Zeno came to the conclusion that the arrow is always at rest during the whole time of its flight.

It is also known that in history there have been numerous attempts to resolve this paradox. The specific solution of it, from the position of dialectics, belongs to Hegel. According to this solution, the cause of Zeno's paradox consists in the one-sidedness (and hence also the metaphysics [i.e., the static character]) of the presentation, that the moving thing in each moment is found in a definite place in space. The truth, however, consists in the fact that at one and the same moment, the thing is and also is not found at a definite place. Therefore, Hegel emphasized, "movement is itself an existing contradiction" (Hegel, *Collected Works*, Vol. 5, Moscow, 1937, p. 521).

By all appearances this solution was recognized in the classics of Marxism-Leninism. For example, the statements about this matter by Engels and Lenin are well known. It is true that in these statements, in all cases very short, either a sort of note or in connection with the analysis of other problems, without explaining whether the phrase "moving thing in one and the same moment is found in a given place and not found in it" is to be a unity of the affirmation and the negation of the very same thing, as Hegel said. Such an interpretation is not infrequently found in the Marxist literature, particularly in the works of authors on dialectical logic.

But in that case a new paradox arises. On one side of the paradox is the said Marxist formula of movement, and the other is the immutable law of formal logic, the law of contradiction. One is incompatible with the other. Here we intend to direct our main attention to this difficulty, rather than taking up a special analysis of movement and its reflection in thought. About this question the recently published essay by S. A. Yanovskii, "Is the difficulty know as 'Zeno's *aporia*' overcome in contemporary science?" (*Problems of Logic*, Moscow, AN USSR, 1963, pp. 116-136) should be recommended.

Authors who recognize correctness of the formula of movement only in the indicated sense naturally turn to the negation of the law of formal logic, and this formula is considered further only a particular case of a general scheme of assertions about dialectically contradictory things and phenomena in reality, corresponding with the law of the unity and struggle of [104] opposites. Therefore this law of Marxist dialectics is opposed to the law of [non-] contradiction. The first is taken in the light of the negation of the second, [the law of non-contradiction]. It is asserted that dialectics prescribes judging things and phenomena, despite the laws of formal logic. True, it is often remarked about this that the law of [non-] contradiction (as well as other laws of formal logic) are not rejected completely. Only their universality is denied (or "absolute universality", although it remains obscure how this differs from simple universality.[1]

However the law of [non-] contradiction, or more precisely, the law of the impermissibility of contradictions in reasoning, which is in question here, says in particular that it is *never* permissible to affirm and deny the very same thing (if it is claimed, certainly, to be the correct solution of some question).

In which cases is it possible and which cases is it impossible judge in opposition to the law of [non-] contradiction?

V. I . Cherkesov asserts that it is impossible to admit "absurd" contradictions, contradictions of "confused reasoning." However, he does not give any criteria for distinguishing such contradictions from "good," permissible ones. E. V. Il'enkov writes "… if contradictions in specific things are manifested with necessity as a result of the movement of thought according to the facts of logic…, then this is not a "logical" contradiction, even if it also has all the formal signs of it, but a completely correct expression of an objective dialectical contradiction"(E. V. Il'enkov, *Dialectics of the Abstract and Concrete* …, p. 241). Thus, if facts lead to the claim A and to the negation of this ~A (the formal symbol of logical contradiction) as well, then so be it. This is not a logical contradiction!

But you see that in any error people make it seems that their thought "was moved by logical factors." Certainly in all sorts of cases of correspondence with all known facts contradictory conclusions have appeared, for example, in physics up to the beginning of the theory of relativity, about the velocity of light depending and not depending on the speed of movement of its source, or the paradoxes of the theory of sets and many others. However, in spite of the opinion expressed, science always considers such contradictions to be logical and subject to removal (resolution), in correspondence with the law of contradiction. Science not only considers them contradictions but also always in the end really resolves them, and often understands at a new level as a result of this.

Another view is also encountered, that it is permissible to claim that an object has some property and also does not have it, but claiming this, it is impossible to negate that same proposition. To put the point differently, the truth of the proposition A &~A is possible, but it cannot be true that (A&~A)&~(A&~A).[2]

It is quite simple (taking into consideration some laws of the algebra of logic, particularly ~~A = A, ~(A&B) = ~A v ~B, A&A = A, AvA = A, A&(BvC) = (A&B)v(A&C), as well as the laws of commutation and association of conjunction) to show that the indicated expressions are equivalent, that is, either both are simultaneously true or both are false:

$$(A\&{\sim}A)\&{\sim}(A\&{\sim}A) = (A\&{\sim}A)\ \&\ ({\sim}A \vee {\sim}{\sim}A) = (A\&{\sim}A)\&({\sim}A \vee A) = ((A\&{\sim}A)\&{\sim}A) \vee ((A\&{\sim}A)\&A) = (A\&{\sim}A) \vee (A\&{\sim}A) = A\&{\sim}A^{3}$$

[105] A wrong view existed for a long time, that it is possible to reason according to the laws of formal logic only in the area of "home use," and this logic itself is declared to be only "beginning", "lowest", "elementary", (see for example: M. N. Alekseev, V. I. Cherkesov, "On the Question of Logic and its study," *Bol'shevik*, 1952, #11). Engels added more to this thought, although with Engels, similar ideas were only established about the principles of metaphysics. These days, after the important role of symbolic logic in scientific cognition was brought out (speaking more exactly, after this role also became known to the representatives of the indicated views), in the form in which formal

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[1] See V. I Cherkesov, *Dialectics as Logic and Theory of Knowledge*, Moscow, MGY Publishers, 1962, p. 314, and also E. V. Il'enkov, *Dialectics of the Abstract and the Concrete in Marx's Capital*, Moscow, AN USSR, 1962, pp. 239-240.

[2] [Here and elsewhere in this paper, the '~' sign has be substituted for the author's indication of negation with an over bar---translator.]

[3] This equivalence is made use of in V. I. Cherkesov's book in order to criticize similar views. However he proves it, referring to the identical falsehood of the expression A&~A, but in his criticizing them (as he himself also does) he does not consider that the expression is always false.

logic appeared in the contemporary stage of development, such a presentation would be an obvious anachronism.

The apparatus of symbolic logic is used thus for the solution of the most complex and precise problems of science, unconditionally exceeding the limits of "home use."[4] The basic systems of symbolic logic (for example, systems with material implication) are such that in them it is proved correct that "anything you wish follows from a contradiction." According to this rule, if in some theory using such a logical apparatus contains a contradiction, then in its logical conclusions are any truth and any falsehood. In other words the limit between truth and falsity completely disappears.

In the light of this the indicated solution to the contradiction between the cited formula of movement and the law of contradiction of formal logic appears, to put it mildly, to be much too daring and hardly admissible for representatives of concrete science. And this, finally, does not provide any benefit to dialectics and also not to science in general. By the way, the power of the law of contradiction is so great that it cannot be contradicted even by the authors mentioned. Coming from their position, they also were obliged to try to resolve the mentioned contradiction. That is why the principle permitting self-contradiction, and the principle that prohibits doing this are not both to be recognized absolutely!

It is essential to note that the presented formula of movement is, perhaps, the only example of a true proposition bearing in mind a formal logical contradiction. In all other cases the enumeration of assertions under this scheme is realized only by more or less obviously strained interpretations. Thus as in other examples, the presence in matter of both discontinuous and continuous features often happens in propositions of physics. However it is difficult to see here the presence of affirmation and negation of the same thing. "Matter is discontinuous and continuous," this, certainly, is not the same as "matter posses the properties of discontinuity and matter does not possess this property." V. I. Cherkesov considered also that despite the law of [non-] contradiction is expressed in the assertion of the presence of north and south poles in a magnet and of the presence in capitalist society of bourgeois and proletariat classes. According to the law of [non-] contradiction the expression A and its negation ~A cannot both be true. However it is possible, V. I. Cherkesov writes, to designate corresponding north and south poles of a magnet or the bourgeoisie and the proletariat, and we get propositions, true ones, despite the law of [non-] contradiction. "These disrupt the conception of the universal significance of the law of [non-] contradiction" (V. I. Cherkesov, *Materialist dialectics as logic and theory of knowledge*, p. 323). However the author omits the view that in the formulation of the law of [non-] contradiction, A and ~A indicate expressions, not objects. A could possible mean, for example, "a magnet has a north pole"; then ~A becomes "the magnet does not have a north pole." Let anyone try to prove that both of these propositions can be true despite the law of [non-] contradiction.

{skipping two pages}

[108] "The term "is found," writes A. Schaff, "in the context that interests us (in the expression "the thing is found in a given place" – E. V.) can have two different meanings to a small degree: "at rest" and "passing by" (Adam Schaff, *op. cit*, p. 349). This train of thought continues this way: The first of these meanings is not suitable for describing movement. If it is cast off then the appearance of the necessity to fall back on contradicting propositions to describe movement disappears, that is, speaking of that moving object as found and simultaneously is not found in a definite point in space. It simply remains to say that a moving object passes by a definite place in each moment. Here there is still nothing contradictory. It is impossible to say that it passes and also does not pass this place at one and the same moment, because only the first clause is true.

If in the expression "a thing is found in a given moment at a give place and not found at that place at that moment," "is found" is understood as "being at rest," then the first conjunct is false; if "is found" means "passes by," then the second conjunct is false.

[4] [Presumably something for "home use" is suitable only for quite limited purposes—translator]

However, in the present case A. Schaff fails to see a third meaning existing here of the term "is found" as a concept related to "being at rest" and "passing by." After all, both here and in other situations there is a general definite relation to the given place. And it is necessary to distinguish this general [relation], if the goal is the analysis of movement. A. Schaff closed himself off from the path to this analysis, adopting the obviously complex concept "to pass by a given place" (in the very formulation of which is already clearly distinguished the above indicated relation to place) as something simple and indefinable through anything else.

"The thing passes by a given place" must obviously mean: 1) it is found at a given place (each point of it has coordinates of a point in space which it passes by) and 2) it has some visible sign differentiating its state of movement from a state of rest. If these are not the exact *differentia specifica*, then what moving thing is not found at the in a given place?

In any cases, it is clear that A. Schaff's proposal about the resolution of the difficulty is not really unimpeachable. "A moving thing passes by a definite place in each moment" – this phrase is still a very bad way to speak about movement, and only states its fact.

K. Aidukevich, on the other hand, does not tend to consider the concept "being found in a given place" as a simple one. He does not consider with Reinakh precisely that this and other concepts are taken without definitions. Primarily he uses the concept "being found at a given moment at a given place" just in the broad sense that A. Schaff disregarded. Through this concept two concepts are defined: "being at rest at a given moment" and "moving at a given moment," which are obviously close to Reinakh's concept "remaining in a given place" and "passing by a given place."

The idea deserves attention, relying on the basic definitions. K. Aidukevich considers that it is impossible to judge about an object that is moved or at rest at a given moment, without considering it before or after this moment. As is known, the application [109] of the method of mathematical analysis for the investigation of movement particularly rests on this idea. Only on account of the behavior of a thing in the neighborhood of a given point can the direction and magnitude of its velocity of movement be defined at a given point, as well as change of its direction, etc.

1. "Object C is at rest at moment of time t" means "there exists an interval of time (t_1,t_2) including t such that in any two moments of time belonging to this interval, the object C is found in one and the same place."

2. "Object C is moving at moment of time t" means "their exists and interval of time (t_1,t_2) containing t such that t does not coincide with its end (i.e., $t_1 < t < t_2$), such that in any two different moments that belong to this interval, the object is found at different places."

In the light of the concepts introduced, writes K. Aidukevich, the following becomes clear. If Zeno asserted that the flying arrow is found in a definite place at each moment of time, having in view by the word "moment" a temporal point lacking duration, then he did not have the basis to conclude that the arrow is found at rest, because, as already explained, in a isolated moment (understood as a temporal point) it is never possible to draw a conclusion about the state of the object. But if by "moment" is meant a temporal interval, even if it is as small as you please, then it would be impossible to speak of the presence of the arrow in each moment at a definite place. From this it follows that Zeno in one case (speaking of finding the arrow at each moment at a definite place) has in view as "moment" a temporal point, and in the other case (concluding that the arrow is a rest at each moment of its flight) he understood "moment" as a temporal interval.

Approaching the analysis of movement from the indicated concepts, it is possible to make a judgment of it without contradiction. However, is this the resolution of this problem? It is easy to imagine that Zeno would be able to answer this objection. "You consider the basic sign of movement to be that at different moments of time a object is found at different places. The flying arrow really is found at different places at different moments—to deny this is impossible. It is possible, finally, to call that state of the object movement according to the definition. Factually this is the appearance of movement, but my interest is the essence of the matter. What in essence is the meaning of your concept of "the object is moving at a given moment?" If a moment is a point that lacks of duration, then how is movement possible in it? Movement of a thing can take place only in an extent of the course of time."

Overall, Aristotle was right in seeing the cause of the paradox in the presentation of time as a collection of moments lacking duration. This is particularly true of the concept of moment as a point of organically connected basic concepts ("found in a given moment at a given place") as only what is considered a mode of analysis of movement. This is incidental about this concept {??}. Representatives of that point of view, for the sake of the refutation that was undertaken in his investigation by K. Aidukevich, consider that a moving object cannot at any moment only be found in a given place, that it is found and also is not found there. Under these conditions it was impossible to take the indicated concept as a basis, unreservedly and without any explanation.

Difficulties connected with the concepts of "moment" also arise if the formula "a moving object at a given moment is found and is not found[5] at a some specific place" is chosen. Is this to be understood in the sense of "moment" as a temporal point without duration, in the sense in which it is used in mathematical writings on mechanical movement in classical mechanics? In mechanics this abstraction is connected with representations of time as a regulated collection of (unmoving) points, but movement as a collection of different situations (places) [110] of things, corresponding to different points of time (see the mentioned essay by S. A. Yanovskii in the collection *Problems of Logic*, p. 133).[6] According to this way of understanding moments it is only possible to say of a moving thing that in each moment it is found in a definite place.

The assertion "found and not found" [in one place] is denied as false. This can also be seen from the fact that it leads to the conclusion that the transition of the thing from one place to another [is regarded] as if it was outside time (if it is recognized that the thing is always found in some place or other). The thing is found in a given moment at place m_1 and is also not found there. But if it is not found in place m_1, then it is found in a different place m_2. However, if it is found at m_2, then this thing (in this same moment!) is not found there. That means it is found in place m_3, etc. Thus in one and the same fixed moment, the thing is found in a series of different places.

Obviously the formula "is found and not found" has in view a moment of real present time, that is, a passing moment. It is not accidental that V. I. Lenin particularly emphasized the unconditionality of moments: "In each given moment … catch this moment " (V. I. Lenin, *Collected Works*, vol. 38, p. 192 [Russian edition])[7]. In such a case it is more exact to say that the moving thing is found at a given moment in a definite place and is *no longer* found there (since the moment is passing). In this the appearance of logical contradiction in the formula of movement disappears. It is difficult, however, to explain what a passing moment of time represents: a small interval of any given size or practically unlimited length (with practically indistinguishable ends). It is also possible to represent it as a point in a moving stream (which is time).[8]

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[5] [Here and elsewhere "found and not found" could also be translated as "present and not present." – translator]

[6] In the opinion of I. S. Narskii, the cause of the antinomy of movement consists in understanding movement only as a series of facts finding material "points" in "point" paths (see *Philosophical Science*, 1964, #1, p. 122). Clearly this claim required some form to be concretized {??}, since the indicated representation of movement in classical mechanics does not lead to contradictions. It is impossible to recognize a well-grounded solution of the question, the proposal in the same number of the journal by B. A. Dragun. The thing, he says, is found in some place and simultaneously in surrounding space. In ….

[7] [The English version of Lenin's works gives the context of this passage as "If one calculates … every second more than ten person in the world die, and still more are born. 'Movement' and 'moment': catch it. At every given moment … catch this moment. Idem in simple mechanical motion (contra Chernov)." Lenin, *Collected Works*, Moscow, 1961, vol. 38, p. 200.--translator]

[8] A moving thing is herewith naturally represented as more or less rigidly connected (depending on the speed of its movement) with this flow and changing coordinates its along with its course. Continuing this analogy, it is possible to say in correspondence with the concepts of the theory of relativity, that the greater the speed of movement of the thing in space, the less the speed of it relative to its own point of time (i.e., the speed change in the time coordinate in Minkowski space).

Other rational interpretations of the formula “found and not found” are possible, under which the word “moment” will be understood as a point in time, but in some sense to be made precise. K. Aidukevich correctly noted that the position of the thing at a moment of time, understood as a point (in the sense of classical mechanics), is undefined – the moving thing cannot possibly differ from the flowing [one]. Between them the problem of the antinomy of movement is illuminated in essence, namely in the search for such a difference (that is not difficult to see in the analysis of its different resolutions).

When mathematical functions are encountered that have an undefined value at some point (for example, sin(x)/x at the point x = 0), then [111] they are investigated it in a neighborhood of this point, reducing that neighborhood in size without limit. The limit that the value of the function as it approaches the point is considered the value of the function at the point. In essence this means that the point itself is considered as the limit of unlimited decrease within its interval. Such an understanding of points is often presupposed in mathematics in place of a previous less well-defined representation. We will approach the analogical form to the clarification of the meaning that is interesting in our opinion. Naturally this will then use other formulations of claims about finding or not finding a thing at a given place. Henceforth it will be convenient for us to speak of finding or not finding a moving thing at a point (or simply a moving point) at a point in space.

We take any point on a moving thing and fix that point in the trajectory of its movement, counting its location at a distance S from some origin and its corresponding point in time t (the end of a interval of time t counted from that beginning). In a given case the concept of point is used in the most common geometrical sense (it is simply a question of giving some significance to the variables S and t).

We take any interval of time (t, t_1), where $t < t_1$, and the interval of the trajectory of movement (S, S_1), where $S < S_1$.

It is permissible that we reduce these unlimited series by removing their ends, but so that the left end (S or t) remains fixed (making, for example, making the interval half as long, then and getting half again from the left, etc.).

Thus we will get an unlimited sequence of time and space intervals, consisting of parts of (t,t_1) and (S,S_1). The temporal point t and the spacial point S will be the limits that the members of these series approach. Let Δt be the variable for the temporal intervals of the indicated set, and ΔS be the variable for the spacial intervals from the second set.

Now the statement 1 “point N is found at moment t at the spacial point S” can be interpreted as:

> 1’. “For any interval ΔS, however small, there can be found a interval of time Δt, in the course of which the point N does not leave ΔS” (i.e., it will be found in ΔS).

If ΔS is diminished without limit, we will come nearer to the point S and still always find the necessary interval Δt. Thus this shows that the moving point can be localized in an interval of space, no matter how small. It is obvious that 1’ in every case is no weaker than 1, since it asserts further the possibility of “retaining” the point N in the course of the time interval (even if it is clear, finally, that in the constriction of ΔS to the point S, Δt also contracts to the temporal point t).

Proposition 2, “the point is not found at the same moment at the point of space S” will mean:

> 2’. “For any interval of time Δt, not matter how small, there can be specified a spacial interval ΔS, from which point N comes out after the time Δt.”

According to the vivid description of H. Reichenbach, “now,” “moment” … slip away the flow of events” (H. Reichenbach, *The Direction of Time*, Moscow, 1962, p. 12). In other places (pp. 35, 65, etc) a moment is defined as a section in time. However a section of the flow, moving along with it, is this one thing, but a section in time as an order set of points is another. The author [Reichenbach] does not discuss these differences.

What is more, by the unlimited diminution of Δt, we will approach the point t, that is, the chosen moment of time, and there is always an interval path, which will pass through point N after this time. Also here 2' is not weaker, judging by everything, than 2.

But 1 and 2 do not contradict one another in the formal-logical sense.

It may be that this is not obvious to everyone. To avoid doubt we can use symbolic language. It is not difficult to see that in 1 and 2 we use only one three-place predicate "point N is found in the course of time Δt within the limits of the interval ΔS". We mark this expression by the symbols $H(N, \Delta t, \Delta S)$.

The negation of this expression (which, obviously, is equivalent [112] to the expression: point N goes out of ΔS after time Δt), which is indicated by $\sim H(N, \Delta t, \Delta S)$. At the same time we note that the concept "found in ΔS in the course of Δt", finally, is clearer than the concept "found in a given moment in a given place," which used as a basis by K. Aidukevich.

Using the symbols we have introduced, and also the symbols $\forall$ ("for all") and $\exists$ ("exists"), we obtain symbolic expression of our propositions.

1'. $\forall \Delta S \exists \Delta t\ H(N, \Delta t, \Delta S)$
2'. $\forall \Delta t \exists \Delta S\ H(N, \Delta t, \Delta S)$

According to the rules of formal logic, in particular the rule for formation of opposites, the opposite of 1' (that is, what is equivalent to its negation) is $\exists \Delta S \forall \Delta t \sim H(N, \Delta t, \Delta S)$. But this expression is not equivalent to 2'. It implies 2' according to the law of predicate logic $\exists x \forall y\ A(x,y) \supset \forall y \exists x\ A(x,y)$, but the reverse does not hold.

Thus it turns out that under a definite interpretation of the expression "a moving thing is found in a given moment in a given place and is also not found in it", it is not a logical contradiction. At the same time, it represents the solution of Zeno's paradox, and is free from the shortcomings of the other solutions examined.

As an example, we take the uniform rectilinear movement, subject to the law $S = Vt$.

For any ΔS it is possible to find a Δt, the existence of which is affirmed by 1'. This will be any interval of time $\leq \Delta S/V$. The same is true for any Δt it is possible to find a ΔS, the existence of which is affirmed by 2'. This will be for any interval $< V\Delta t$.

It is not difficult to show that 1' and 2' assert as well the continuity of the size of the path S as a function of t and the continuity of t as a function of the size of the path in any moment of time and at any point of space. In the matter itself the continuity of the function f(x) at some point x means that everywhere is it possible to find an increase in the argument which corresponds to an increase in the function which does not exceed the given value. But 1' just also affirms that for any ΔS it is possible to find a Δt for which the increase in the path at this time does not exceed ΔS. Similarly, 2' asserts that for any Δt there can be found an increase in the path ΔS for which the time that is necessary for the transiting this path (that is, the increase in time, considered as a function of the path) proves to be less that Δt.

Consequently, choosing any interval $\Delta_1 S$ from the set of intervals indicated above, it is possible to find (according to 1') an interval of time $\Delta_1 t$ in the course of which the point N does not leave $\Delta_1 S$; but for this $\Delta_1 t$ it is possible to find (according to 2') a $\Delta_2 S$ from which the point N comes out after time $\Delta_1 t$. Then for $\Delta_2 S$ is found (according to 1') a $\Delta_2 t$, in the course of which N does not leave $\Delta_2 S$, etc.

Here two mutually excluding processes are interwoven: from one side, unlimited division of time, the result of which there is always found a time at which a moving point cannot pass some barrier (in itself this would make movement impossible), as a result of which is always found an interval within whose limits the thing still does not remain for this whole time. Movement is also the constant resolution of this dialectical contradiction.